Characterisation of the robustness of weighted networks, a first step to better understand the context of humanitarian operations

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Abstract In situations where crises are succeeding one to another, it is important to understand and measure the strengths and weaknesses of one's logistics network. This observation applies to many companies. It is even more relevant for humanitarian organizations, who are confronted to increased demand for humanitarian aid without having a sufficient budget to cover all present and future needs. Our proposal allows to visualize these strategic points, using complex networks. We measure the robustness of local infrastructures (health and logistics) by simulating its response in the event of a crisis. Non binary attacks, where nodes and/or links are damaged but not removed entirely are used in order to remain as close as possible to the real phenomenon, where the damages suffered by infrastructures may hinder their capacity but not always totally destroy it. We also use weighted networks. This work is carried out in close collaboration with Handicap International, so as to validate the relevance of the approach and its applicability through real applications.

Keywords Complex Systems · Robustness · Percolation · Humanitarian Logistics

1 Non Binary Percolation

According to Bellingeri et al., assessing the robustness of real-world systems with a percolation threshold is relevant only if at least two assumptions are

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verified. Firstly, during the percolation process, the binary (i.e. unweighted) modeling of the network should not over simplify the system. Secondly, the binarity of the theoretical attack (spared or deleted) should match with the actual pressure [2]. However, some complex real-world systems do not satisfy these assumptions. Networks are not only specified by their topology but also by the dynamics of information or traffic flow taking place on the structure [1]. In particular, the intensity of connections may be very important in the understanding of some systems. For example, in a transportation network, roads have different traffic capacity, a highway will be much busier than a trunk road. Here assumption (i) would not be verified. Plus, in real-world context, the articles [7, 3, 2] warn that a system can collapse long before the connections between the elements are broken. In other worlds, accidents or traffic jams could seriously slow down traffic while the roads remain open.

This summary proposes an approach that we call the non binary percolation (NBP) to evaluate the robustness of networks. This theory is in line with the percolation but it allows to take into account the weighting of networks and the resistance of elements under pressure. We consider that the weights represent a flow, a service or an exchange capacity. When a system is under pressure, its elements struggle to remain functional but do not systematically collapse. This observation is modeled by a non-binary attack. Elements can be spared or deleted, but also damaged, and therefore less effective.

2 Robustness Indicators

Xing Pan and Huixiong Wang highlight that the conclusions about robustness depend to a large extent on the chosen indicators [8]. Bellingeri et al. establishe that deleting a very small fraction of selected edges does not necessarily affect the largest connected component (LCC) but it can produce a rapid collapse of weighted condition measures [2]. Consequently, it is of primary importance to choose the right health indicator.

We consider four potential robustness indicators :

***S*.** Usual percolation uses S (the fraction of nodes belonging to the LCC) to evaluate the state of the network and defines the robustness by the percolation threshold p_c. It is commonly used, but it evaluates only the topological connectedness of the network, and neglects the weights [2, 8].

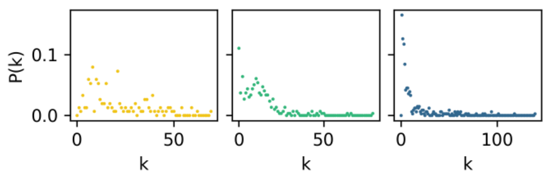
***Eff*.** Efficiency allows a precise analysis of the information flow on weighted networks and informs on the average distance between nodes [5].

Eff_{thr}. Similarly to percolation threshold, we define Eff_{thr} as the smallest p such that Eff is not zero. When the attack is sufficiently destructive, the simulation shows a phase transition at point Eff_{thr} exactly as in percolation.

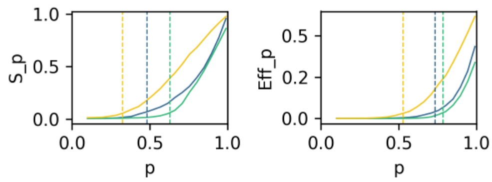
Eff_{int}. As proposed in [6] in a classical percolation context, we suggest to use the area under the curve of the Eff during the NBP process. The larger the area, the greater the Eff during the process. This measure allows to hierarchize the robustness of the networks regardless of the Eff_{thr} value and reflects the overall behavior of the network during all of the attacks.

Name	N	L	E	V	Eff	Eff_{thr}	Eff_{int}	S_{thr}	p_c
USAir97	332	2126	0.14	0.01	0.43	0.73	0.05	0.37	0.02
C. elegans	297	1720	0.05	0.01	0.34	0.78	0.04	0.52	0.05
Dolphin socialisation	151	1554	0.24	0.01	0.62	0.53	0.13	0.21	0.03

(a)



(b)



(c)

Fig. 1 (a) Statistics on reals networks (b) Degree distributions. (c) NBP process. Legend : Dolphin socialisation (yellow), C. elegans (green) and USAir97 (blue).

3 Simulation on Real networks

In this section, we apply our approach to real networks : **USAir97**, which describes air routes between American airports, **C. elegans**, which describes the worm brain network [4], and **Dolphin socialisation**, which is the result of 124 days of dolphin watching in the Cedar Key area, Florida.

The usual percolation gives the following theoretical thresholds 0.02, 0.03 and 0.05 for respectively USAir97, Dolphin socialisation and finally C. elegans. NBP delivers more contrasting and different scores. The ranking is not the same either, as illustrated in figure 1(a). We observe a phase transition of Eff_{int}. Figure 1(c) reveals that Dolphin socialisation are much more robust with $Eff_{int} = 0.13$, then USAir97 with 0.05, and C. elegans with 0.04. We can see, especially for the dolphin socialisation network, that considering the condition of a network by its topology leads to an overestimation of its health.

With those robustness indicators, we intend to measure the robustness of huanitarian response networks, such as the one presented by Figure 2.

4 Conclusion

We propose to perform a non binary percolation to measure the robustness of existing networks if a large-scale natural crisis occurs. Different indicators of robustness are possible here. We can cite the classic indicator, S, which is calculated by determining the fraction of the remaining nodes located in the giant component. Other indicators related to the efficiency of the system, its ability to transmit product flows even when damaged are being studied. This summary focuses on various possibilities to measure the robustness of the networks. It is a preliminary study, before application to real humanitarian logistics networks. The possibilities offered by network theory are numerous and make it possible to better exploit the available datasets in order to improve the crisis response process.

Fig. 2 Force Atlas representation of the network during relief operation in Haiti after both the earthquake and the cholera outbreak, end of 2010

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